Fermionic QW Coupled to a Bosonic Reservoir*

Alain Joye

*joint with O. Bourget, D. Spehner, AHP '25

Quantissima/Oise 15-19/9/25

Quantum Walk(er)

Graph: $\mathscr{G}$ with vertices $\{x\}_{x\in v}$

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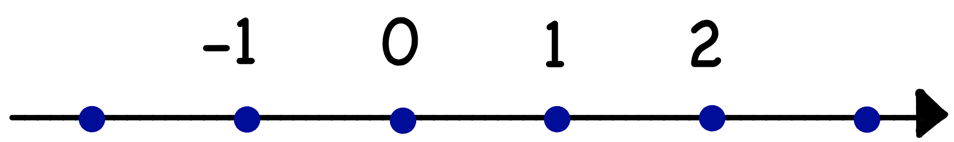
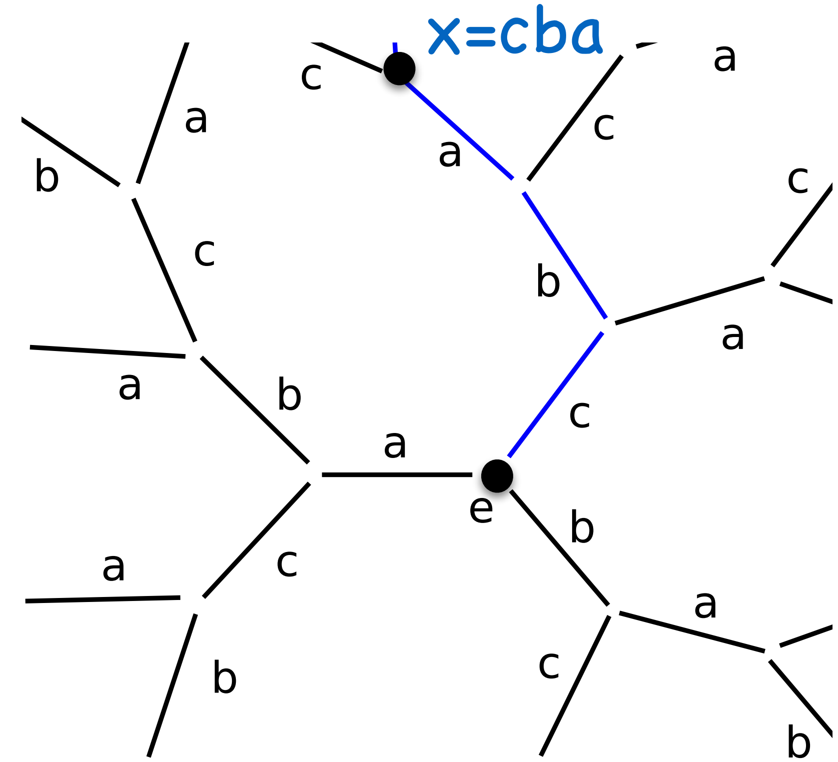
e.g.



$\mathbb{Z}$



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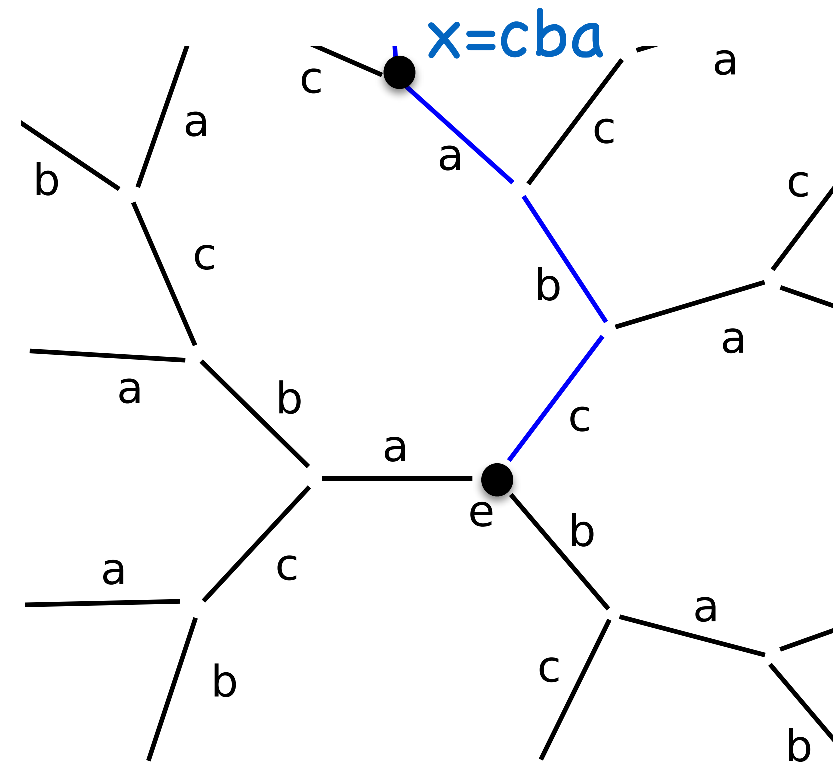

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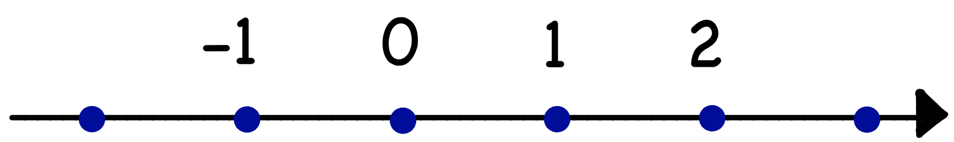
e.g. 

$\mathbb{Z}$

Hilbert space:

$\mathscr{H}_{\mathscr{G}} = l^2(\mathscr{G})$ with ONB $\{e_x\}_{x\in v}$ & scal. pdt. $\langle \cdot | \cdot \rangle$

s.t. $l^2(\mathscr{G}) \ni \psi = \sum_{x\in v} \psi_x e_x \,, \quad \psi_x \in \mathbb{C}\,, \quad \sum_{x\in v} |\psi_x|^2 < \infty$

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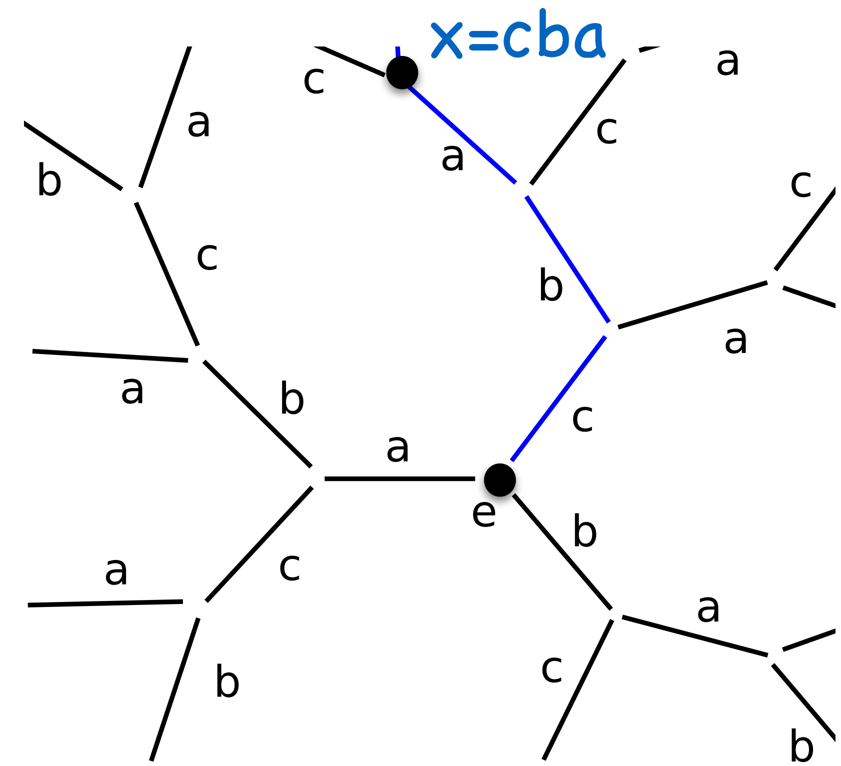

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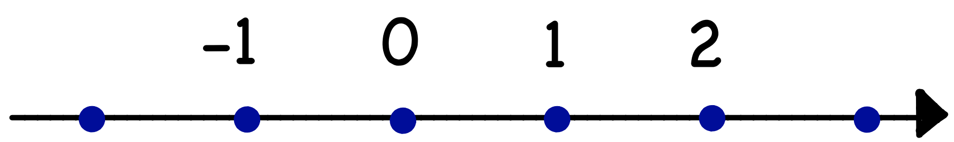

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Unitary operator: V on $\mathscr{H}_{\mathscr{G}} = l^2(\mathscr{G})$

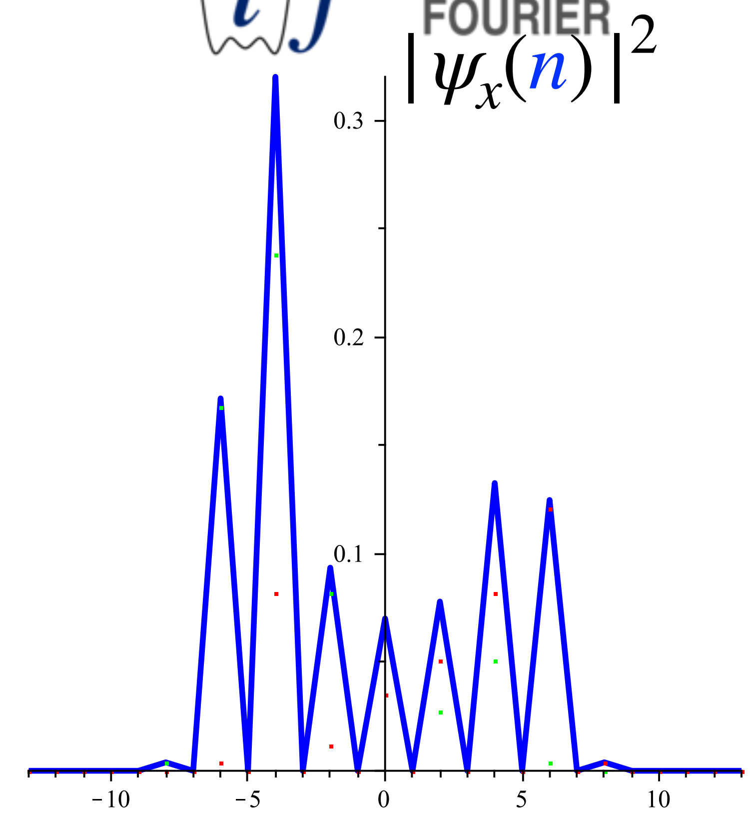
s.t. $\quad \langle e_x | V e_y \rangle = 0$ if $\mathrm{dist}_{\mathscr{G}}(x,y) > D$ for $\mathscr{G}$ infinite

Quantum Walk(er)

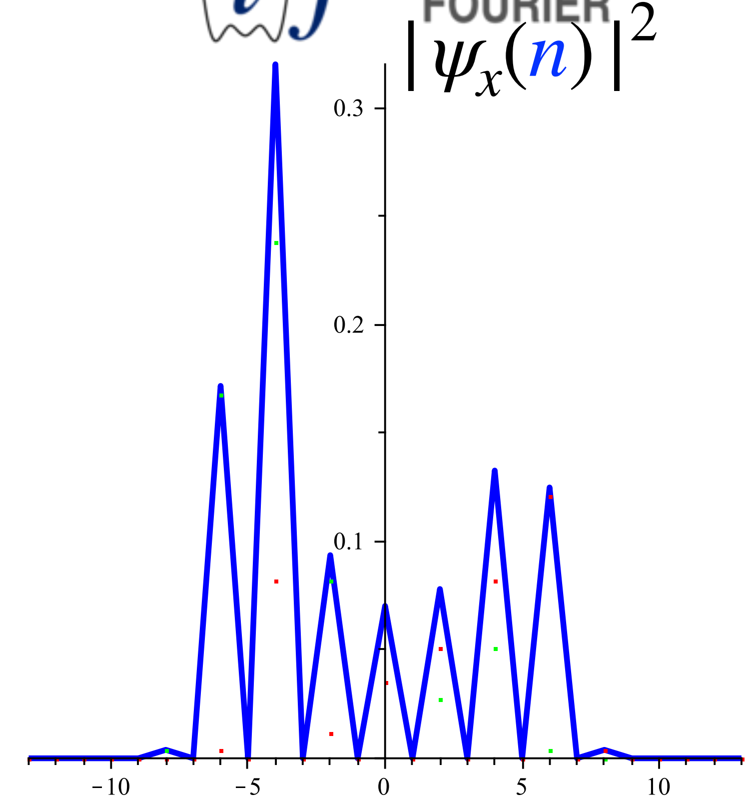
Discrete dynamics:

time $t = 0$ $\qquad \psi(0) = e_{x_0} \in \mathscr{H}$

$t \in \mathbb{Z}$ $\qquad \psi(t) = V^t \psi(0) = \sum_{x \in v} \psi_x(t) e_x$



Quantum Walk(er)



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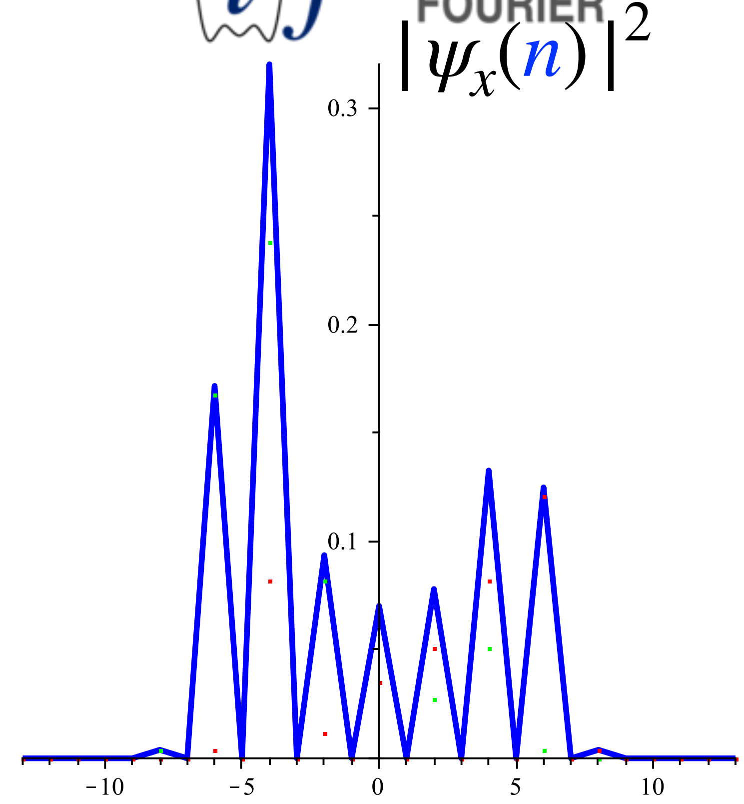
Exples:

On $\mathbb{Z}$, the shift $S : \delta_j \mapsto \delta_{j-1}\,,\ \ j \in \mathbb{Z}$

On $\mathscr{G}$ finite, any V unitary

Quantum Walk(er)



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- Approximate dynamics of quantum systems

Feynman'82, Chalker-Coddington'88, Meyer'96,..., Manouchehri-Wang'03

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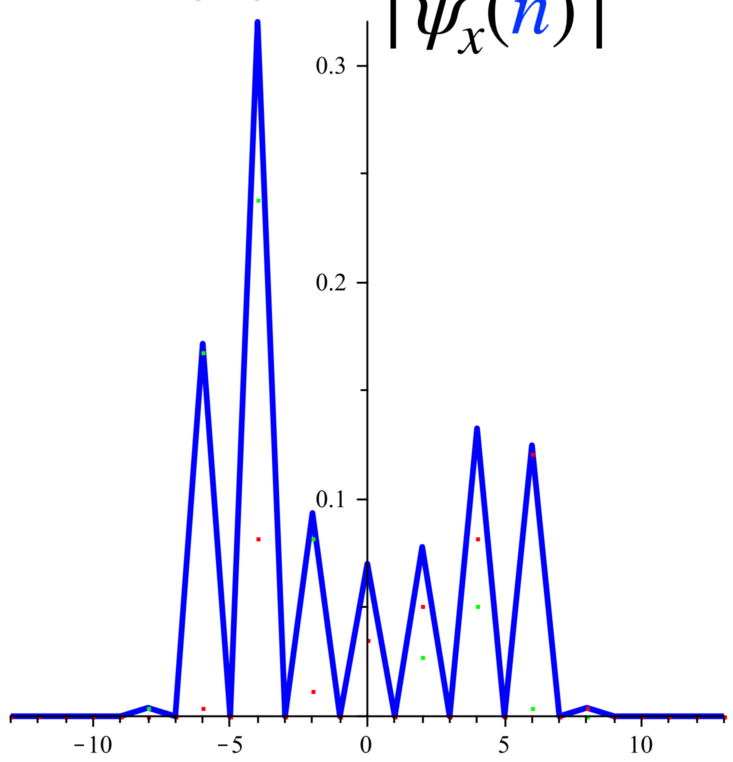

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- Quantum software for quantum computer

Aharonov et al'93, Grover'96, Childs et al'03,...

"Thermalization" Problem

Problem:

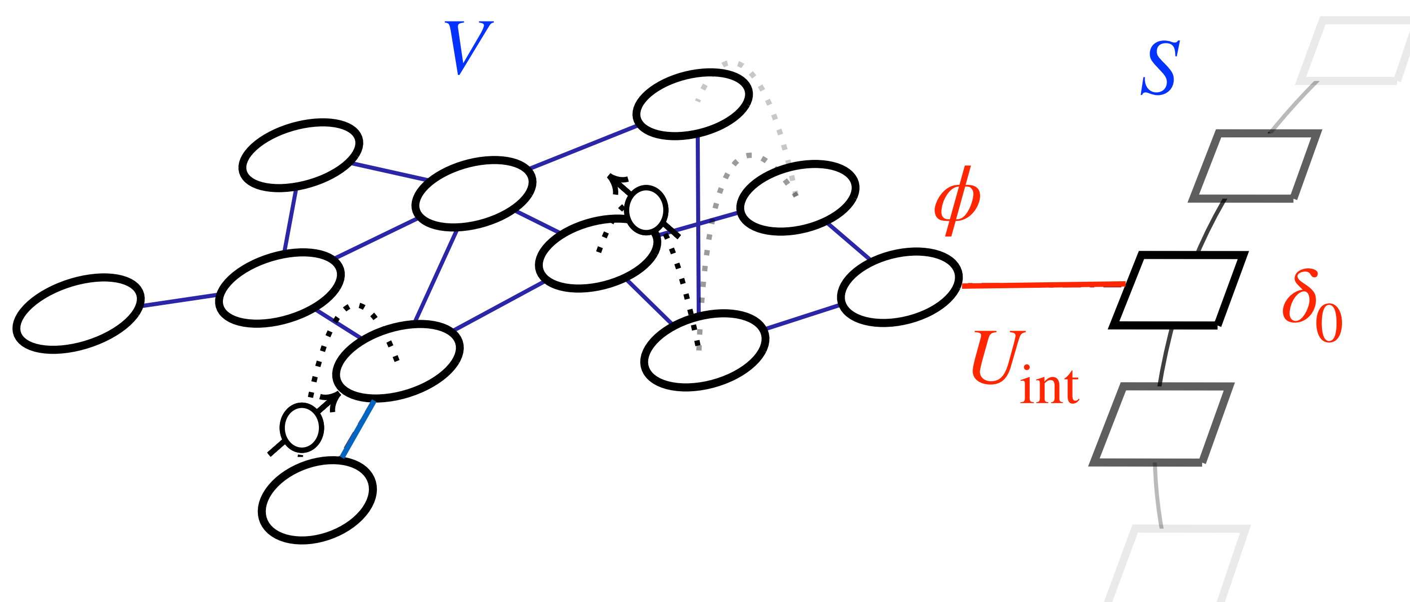
On $\mathscr{G}$ finite, any V unitary

reservoir

bosons

sample

fermions



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Hamza, J. '17, Raquépas '20, Andréys, J., Raquépas '21

Fermionic QWs

Fermions: antisym. subsp.

$\rightsquigarrow \mathscr{H}^{\wedge N}$ spanned by $\{e_{x_1} \wedge e_{x_2} \wedge \ldots \wedge e_{x_N}\}_{x_i \in v, 1 \leq i \leq N}$

s.t. $e_{x_1} \wedge e_{x_2} \wedge \ldots \wedge e_{x_N} = \mathrm{sign}(\sigma) e_{x_{\sigma(1)}} \wedge e_{x_{\sigma(2)}} \wedge \ldots \wedge e_{x_{\sigma(N)}}$

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Fock space: $\dim \mathscr{H} = d$

$\mathscr{F}_- = \oplus_{0 \leq N \leq d} \mathscr{H}^{\wedge N}$ s.t. $\psi = (\psi_0, \psi_1, \psi_2, \cdots, \psi_d)$ with $\psi_N \in \mathscr{H}^{\wedge N}$

$\Psi_0 \in \mathbb{C}\Omega$, Ω the vacuum

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$\Psi_0 \in \mathbb{C}\Omega$, $\quad \Omega$ the vacuum

Non interact. Dyn. on $\mathscr{F}_-$:

V on $\mathscr{H} \rightsquigarrow \Gamma_-(V)$ s.t. $\quad \Gamma_-(V)|_{\mathscr{H}^{\wedge N}} = V \wedge V \wedge \cdots \wedge V$

Creation/annihilation op.

Set $a^*(f) : \mathscr{H}^{\wedge N} \to \mathscr{H}^{\wedge(N+1)}$ s.t. $a^*(f)\Omega = f$ $\forall f, f_1, f_2, \ldots, f_N \in \mathscr{H}$

$$a^*(f)(f_1 \wedge f_2 \wedge \ldots \wedge f_N) = f \wedge f_1 \wedge f_2 \wedge \ldots \wedge f_N$$

and $a(f) : \mathscr{H}^{\wedge N} \to \mathscr{H}^{\wedge(N-1)}$ s.t. $a(f)\Omega = 0$ by $a(f) = (a^*(f))^*$

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Rem: $\Gamma\_(e^{iH}) = e^{id\Gamma\_(H)}$ $\quad d\Gamma\_(H) = \sum_{i,j} \langle e_i \,|\, He_j\rangle a^*(e_i) a(e_j)$ ONB $\{e_j\}_j$

Bosonic QWs

Bosons: sym. subsp.

$\rightsquigarrow$ $\mathscr{H}^{\otimes_s N}$ spanned by $\{e_{x_1} \otimes_s e_{x_2} \otimes_s \ldots \otimes_s e_{x_N}\}_{x_i \in v, 1 \leq i \leq N}$

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Fock space:

$\mathscr{F}_+ = \oplus_{0 \le N} \mathscr{H}^{\otimes_S N}$ s.t. $\psi = (\psi_0, \psi_1, \psi_2, \cdots)$ with $\psi_N \in \mathscr{H}^{\otimes_s}$

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$$b^*(f)(f_1 \otimes_s f_2 \otimes_s \dots \otimes_s f_N) = \sqrt{N+1} f \otimes_s f_1 \otimes_s f_2 \otimes_s \dots \otimes_s f_N$$

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CCR: $[b(f), b^*(g)] = \langle f | g \rangle \mathbb{1}, \qquad [b^\#(f), b^\#(g)] = 0 \qquad \forall f, g \in \mathscr{H}$

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Weyl op.: $W(f) = e^{i(b(f)+b^*(f))/\sqrt{2}}, \ f \in \mathscr{H}$ unitary

s.t. $W(f)W(g) = W(f+g)e^{-i\mathrm{Im}\langle f|g\rangle/2}$

Quantum States

State on $\mathscr{F}_-$:

$$\mathscr{D} = \{\rho \in \mathscr{T}(\mathscr{F}_-) \text{ s.t. } \rho \geq 0,\ \mathrm{tr}\,\rho = 1\}$$ density mat.

s.t. for $X = X^* \in \mathscr{B}(\mathscr{F}_-)$ observable

$$\omega_\rho(X) = \mathrm{tr}(\rho X) \in \mathbb{R}$$ QM expect. value

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Gaussian State on $\mathscr{F}_+$:

$\omega_K \leftrightarrow$ symbol $K \in \mathscr{B}(l^2(\mathbb{Z}))$, $K = K^* \geq \mathbb{I}$ s.t.

$$\omega_K(W(f)) = \exp\big(-\langle f \,|\, Kf\rangle/4\big), \quad f \in l^2(\mathbb{Z})$$

$\rightsquigarrow$ $\omega_K(A) \in \mathbb{R}$ the QM expect. val. of $A \in \mathrm{CRR}(l^2(\mathbb{Z}))$

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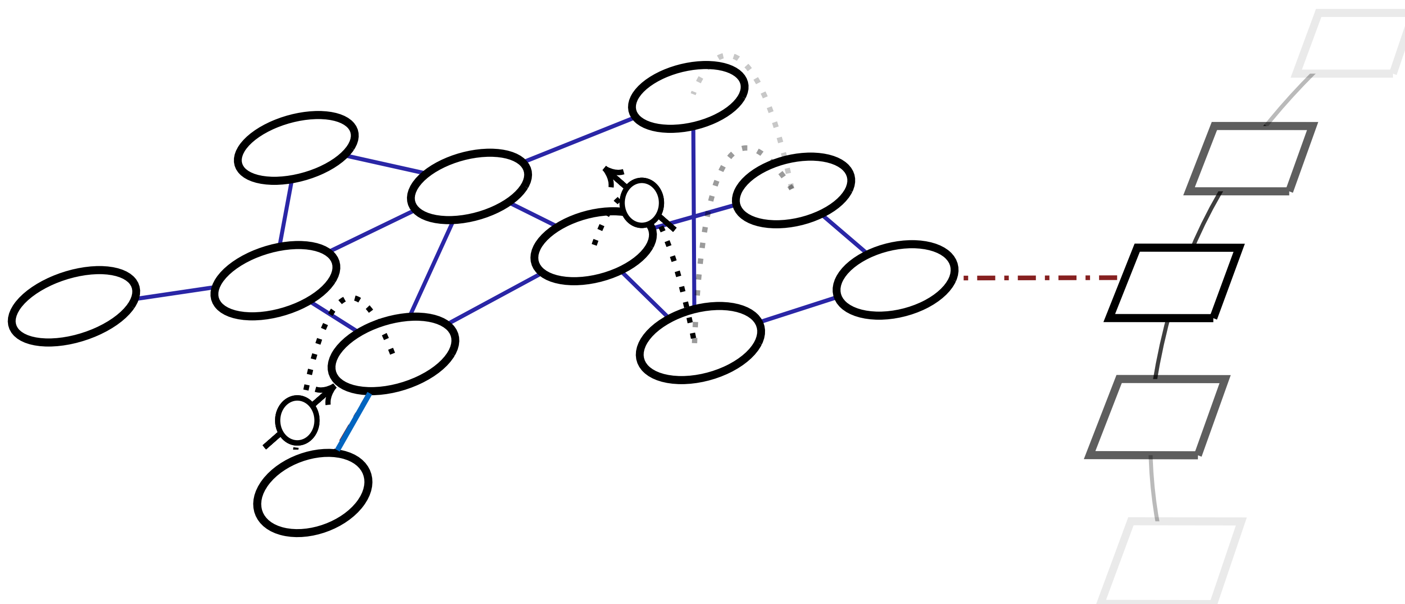
Exple: Thermal case

$$K = \coth\big(\beta(H_B - \mu\mathbb{I})\big) \quad \text{for} \quad H_B = H_B^* > \mu\mathbb{I},\ \beta > 0$$

Model

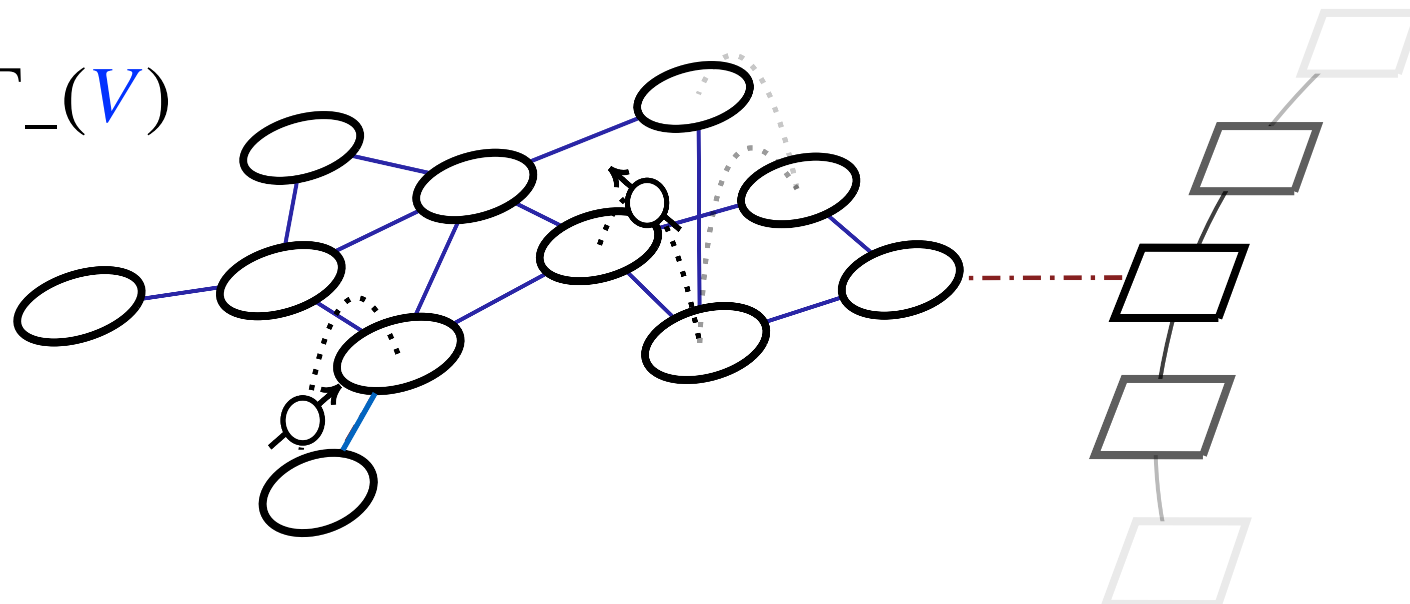


Model

$\Gamma\_(V)$

fermionic

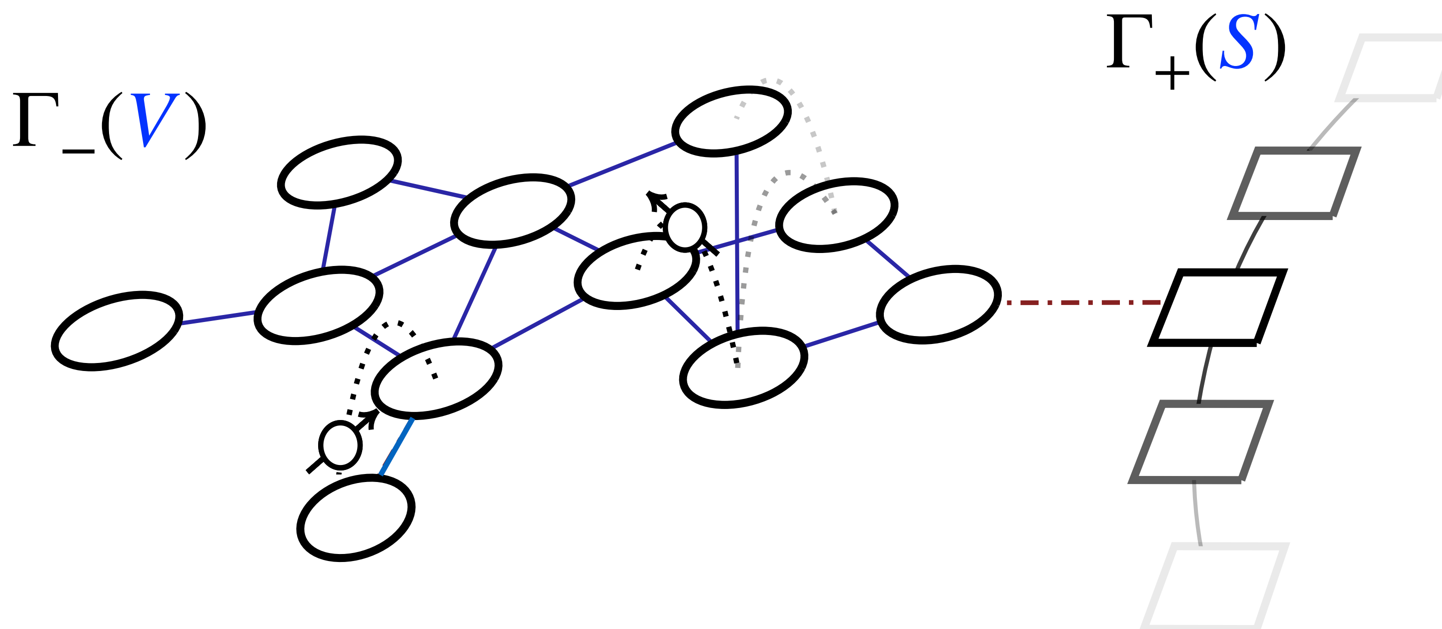
sample



Finite Graph: $\mathscr{G}$ $\rightsquigarrow$ sample with dyn. $\Gamma\_(V)$ on $\mathscr{F}\_(\mathscr{H}_{\mathscr{G}})$

Model

fermionic sample

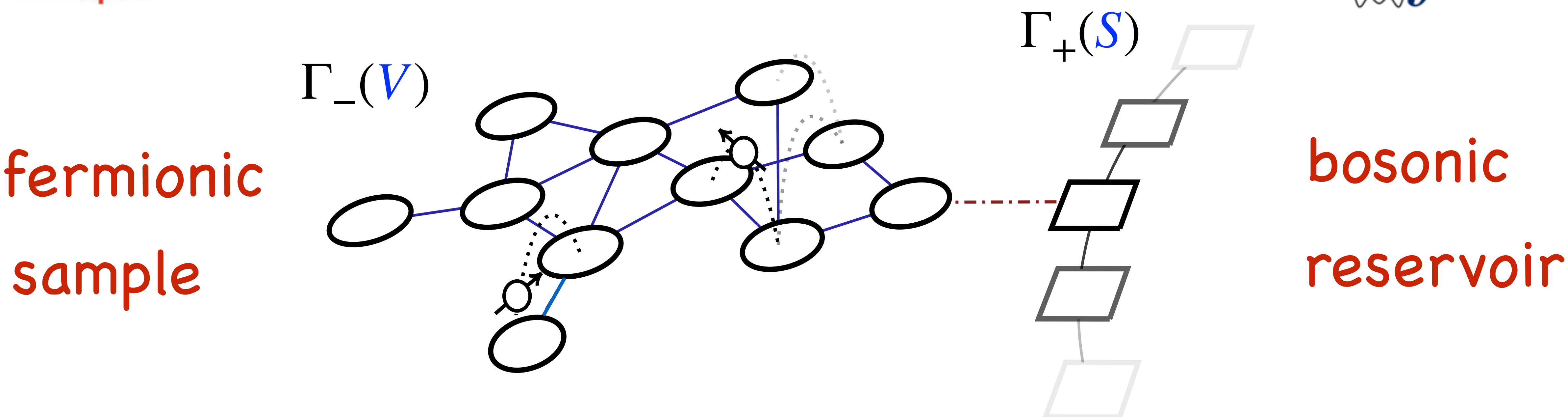


bosonic reservoir

Finite Graph: $\mathscr{G}$ $\rightsquigarrow$ sample with dyn. $\Gamma_-(V)$ on $\mathscr{F}_-(\mathscr{H}_{\mathscr{G}})$

Infinite Graph: $\mathbb{Z}$ $\rightsquigarrow$ reservoir with dyn. $\Gamma_+(S)$ on $\mathscr{F}_+(l^2(\mathbb{Z}))$

Model



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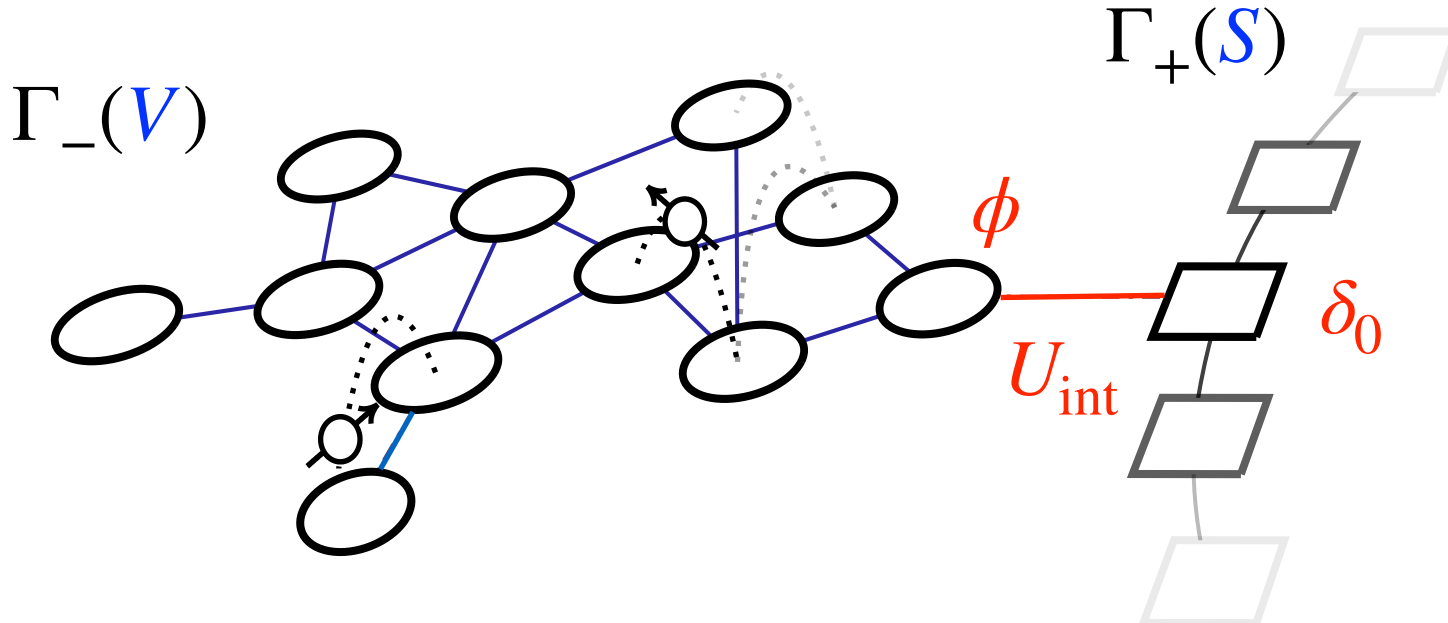
Total Hilbert space: $\mathscr{F}_-(l^2(\mathscr{G})) \otimes \mathscr{F}_+(l^2(\mathbb{Z}))$

Model

fermionic sample



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Total Hilbert space: $\mathscr{F}_-(l^2(\mathscr{G})) \otimes \mathscr{F}_+(l^2(\mathbb{Z}))$

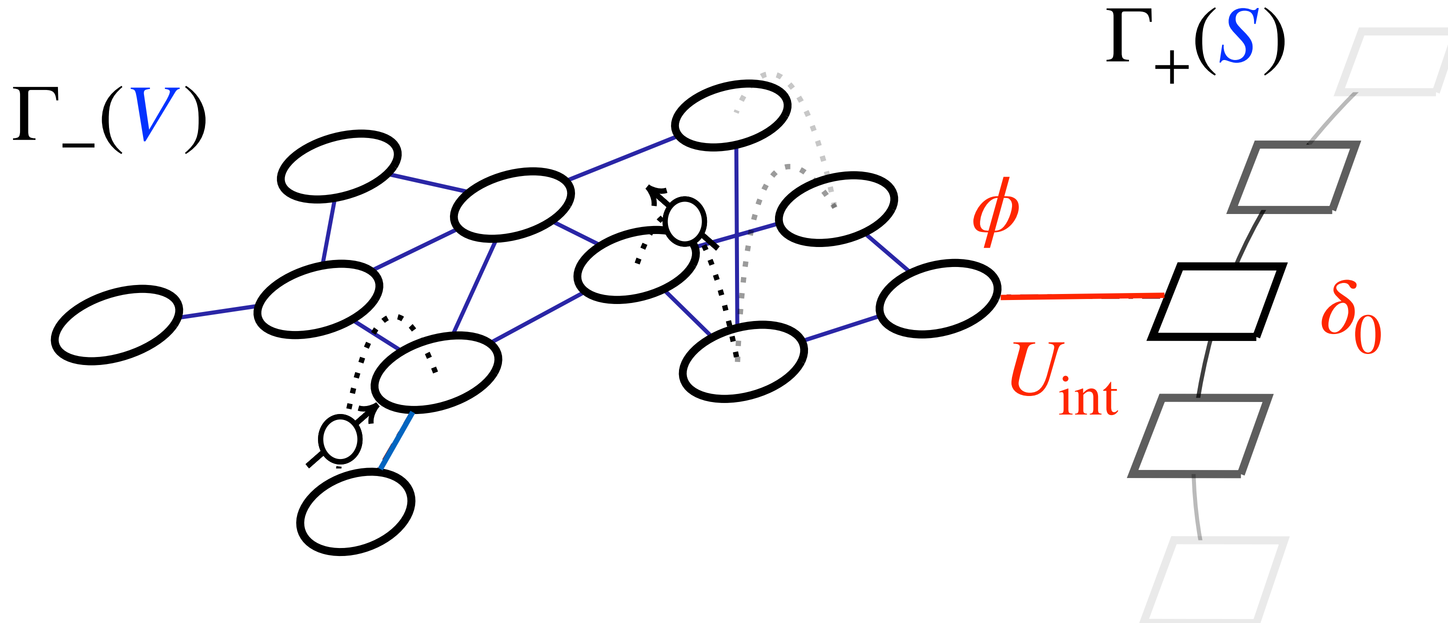
Unitary Coupling: $U_{\text{int}} = e^{-i\lambda T \otimes (b_0 + b_0^*)/\sqrt{2}}$, λ coupling strength

$T = T^* \in \mathscr{B}(\mathscr{F}_-)$, $b_0^\# = b^\#(\delta_0)$

Model

fermionic

sample



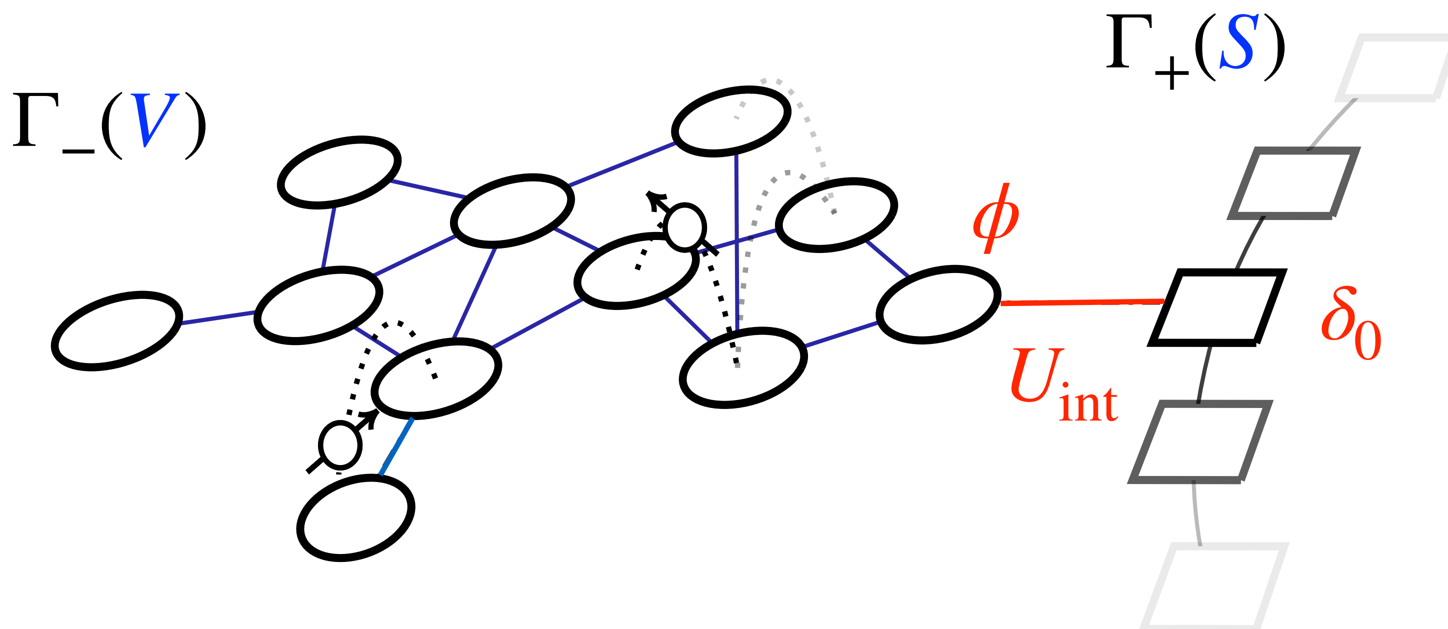
bosonic

Gaussian

reservoir

Model

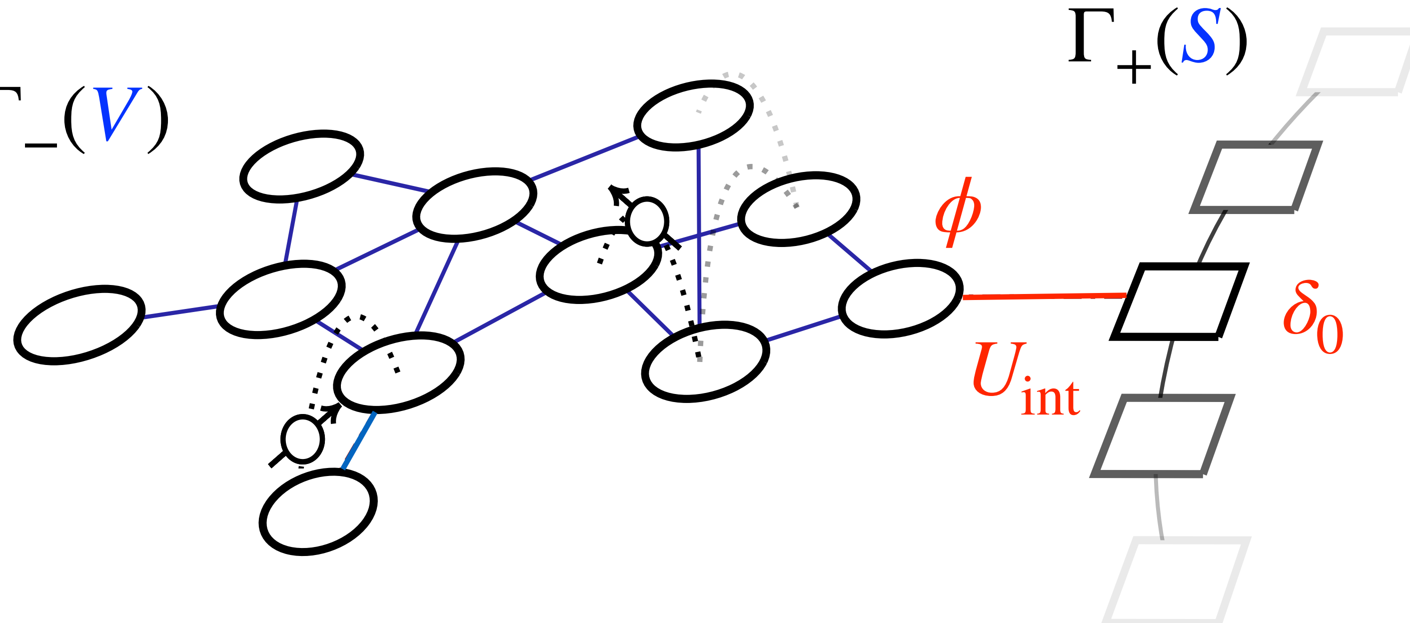
fermionic sample



bosonic Gaussian reservoir

Unitary dynamics: $U_\lambda = U_{\text{int}}(\Gamma_-(V) \otimes \Gamma_+(S))$

Model



fermionic sample

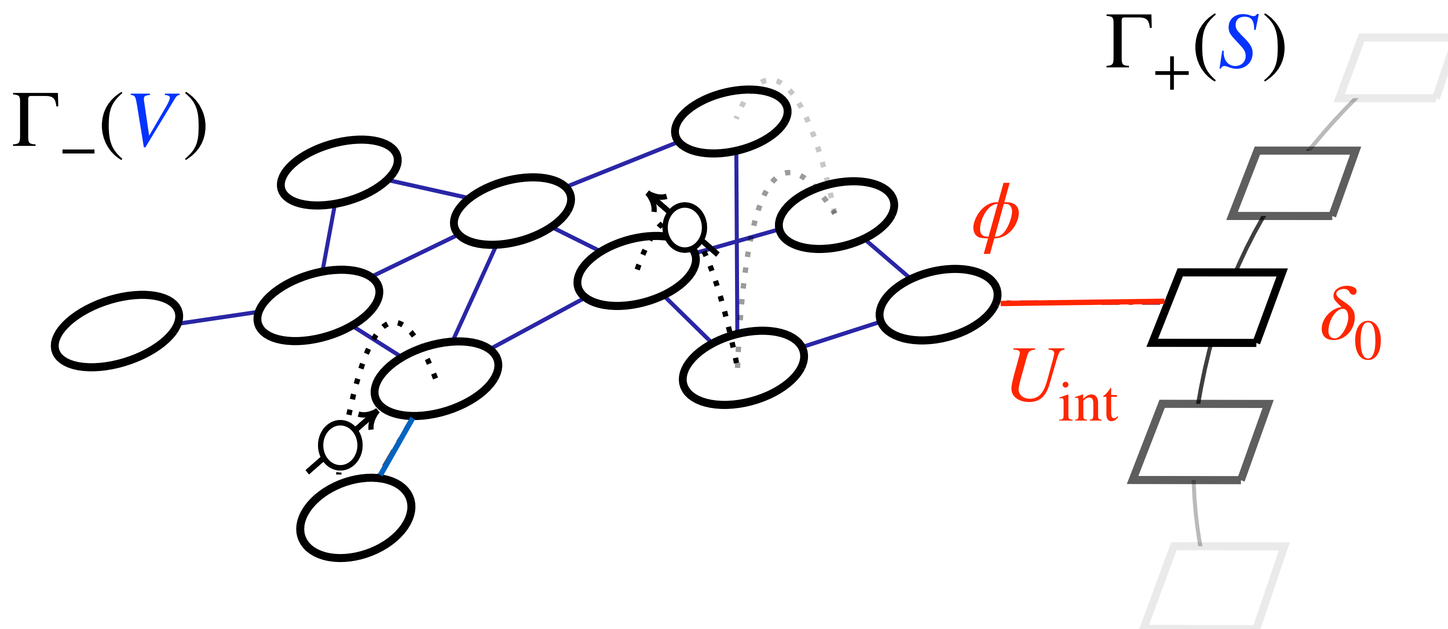
bosonic Gaussian reservoir

Unitary dynamics: $U_\lambda = U_{\rm int}(\Gamma_-(V) \otimes \Gamma_+(S))$

State: $\omega_\rho \otimes \omega_K(\,\cdot\,)$ at time $t = 0$

Model

fermionic sample



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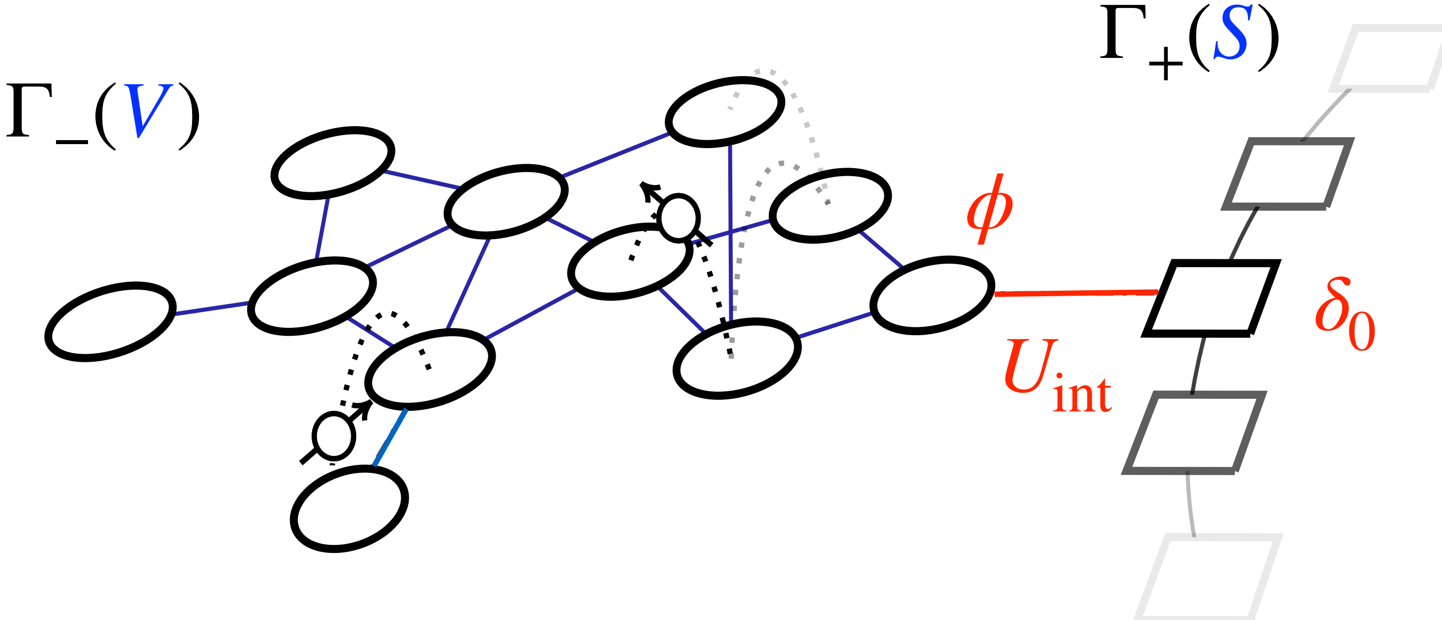
$\omega_\rho \otimes \omega_K \circ \mathscr{U}_\lambda(\,\cdot\,)^t$, $\mathscr{U}_\lambda(\,\cdot\,) = U_\lambda^* \cdot U_\lambda$ at time $t \in \mathbb{N}$

Model



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$\omega_\rho \otimes \omega_K \circ \mathscr{U}_\lambda(\,\cdot\,)^t$, $\mathscr{U}_\lambda(\,\cdot\,) = U_\lambda^* \cdot U_\lambda$ at time $t \in \mathbb{N}$

Effective dyn. on sample: $\mathscr{T}_t : \mathscr{B}(\mathscr{F}_-) \to \mathscr{B}(\mathscr{F}_-)$, $t \in \mathbb{N}$

s.t. $\omega_\rho(\mathscr{T}_t(X)) = \omega_\rho \otimes \omega_K \circ \mathscr{U}_\lambda^t(X \otimes \mathbb{I})$ $\forall X \in \mathscr{B}(\mathscr{F}_-), \rho \in \mathscr{D}$

Effective dynamics

Interaction: $U_{\mathrm{int}} = e^{-i\lambda T\otimes(b_0+b_0^*)/\sqrt{2}}$, $T = T^* \in \mathscr{B}(\mathscr{F}_-)$, $b_0^\# = b^\#(\delta_0)$

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$T = \sum_{\mu \in \sigma(T)} \mu B^\mu$, e.-val. $\mu \in \mathbb{R} \leftrightarrow B^\mu \in \mathscr{B}(\mathscr{F}_-)$ e.-proj.

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Weyl op.

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Proposition: $\mathscr{T}_t : \mathscr{B}(\mathscr{F}_-) \to \mathscr{B}(\mathscr{F}_-), \ \ t \in \mathbb{N}$

- $$\mathscr{T}_t(X) = \sum_{\underline{\mu},\underline{\nu}\in\sigma(T)^t} B_1^{\mu_1}\cdots B_t^{\mu_t}\mathscr{V}^t(X)B_t^{\nu_t}\cdots B_1^{\nu_1}\exp\left\{-\frac{\lambda^2}{4}\left\langle\Theta_{\underline{t}}^{\underline{\mu}-\underline{\nu}}, K\Theta_{\underline{t}}^{\underline{\mu}-\underline{\nu}}\right\rangle\right\}$$

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Bosonic weight

$K \geq \mathbb{0}$

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$\rightsquigarrow$ large coupling exp.

Spectral approach

$$\Rightarrow \quad \mathscr{T}_{\color{blue}t}\big|_{\color{red}\lambda\color{black}=\infty}(X) = \sum_{\underline{\mu}\in\sigma(T)^t} B_1^{\mu_1}\cdots B_t^{\mu_t}\mathscr{V}^t(X)B_t^{\mu_t}\cdots B_1^{\mu_1} = (\mathscr{V}\Phi)^{\color{blue}t}(X),$$

with $\quad \Phi(\,\cdot\,) = \sum_{\mu\in\sigma(T)} B^\mu \cdot B^\mu \qquad \& \qquad \mathscr{V}(\,\cdot\,) = \Gamma\_(V^{-1})\cdot\Gamma\_(V)$

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Φ Orthog. proj.

$\mathscr{V}$ Unitary op.

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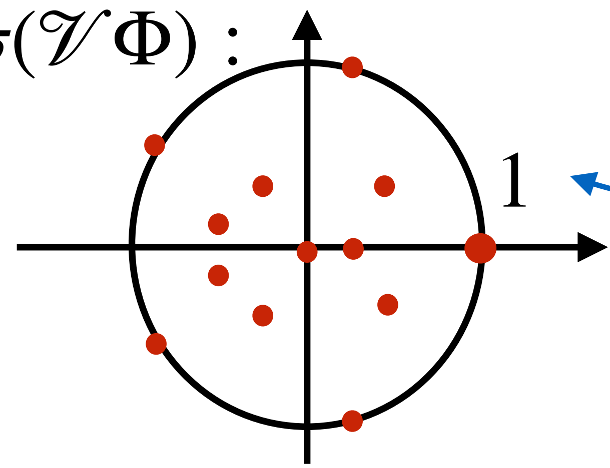

$\mathscr{T}_1\big|_{\lambda=\infty}$ unital, CP

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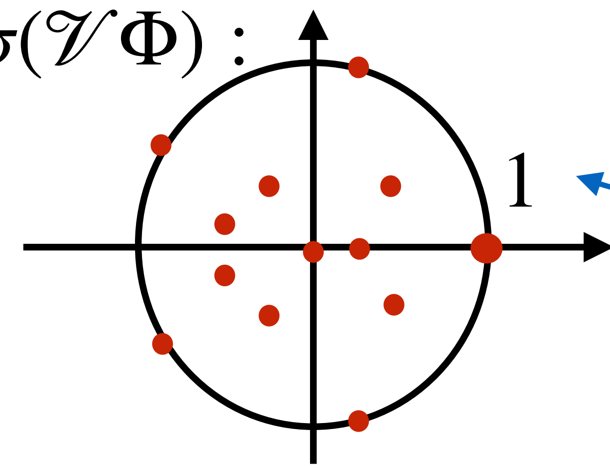

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$$\mathcal{V}\Phi = \sum_k \lambda_k \mathcal{P}_{\lambda_k} + \mathcal{N}_{\lambda_k}$$ Spec. dec.

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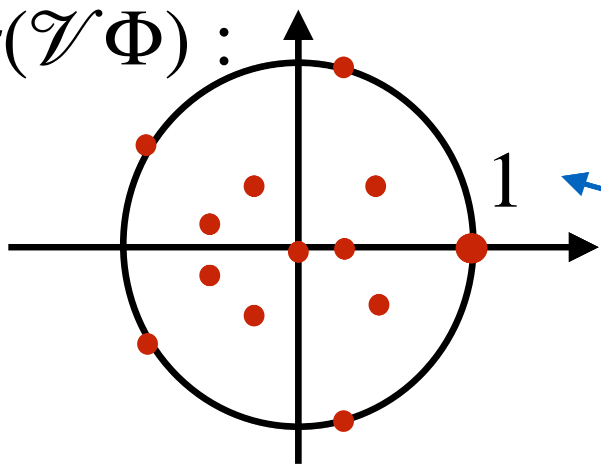

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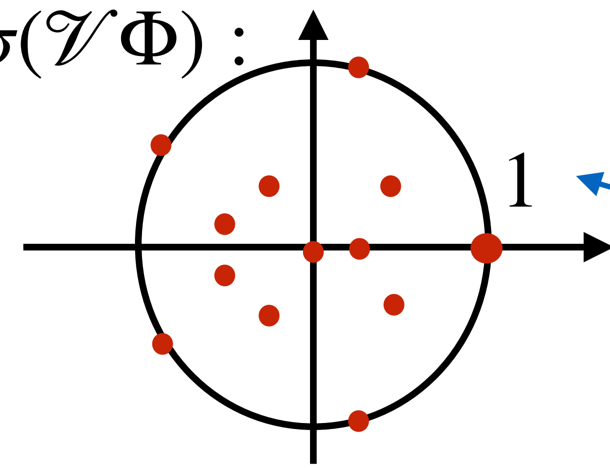

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Large coupling approx.

Thm: Let $\Delta = \min_{\substack{\mu,\nu\in\sigma(T)\\ \mu\neq\nu}} |\mu-\nu|, \qquad \gamma < \min_{|\lambda_k|<1} |\ln|\lambda_k||.$

Then $\exists\lambda_0>0$ s.t. for all $\lambda>\lambda_0$, $t\in\mathbb{N}$,

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Let $\tau = \sum_{1 \leq j \leq d} \epsilon_j |f_j\rangle\langle f_j| \in \mathscr{B}(\mathscr{H}_{\mathscr{G}})$ a one body obs.

Set $T = d\Gamma_{-}(\tau) = \sum_j \epsilon_j a^*(f_j) a(f_j) \in \mathscr{B}(\mathscr{F}_{-})$ 2^{nd} quantization

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Consequence:

The op. $\mathscr{T}_t, \mathscr{V}, \Phi$ leave $\mathscr{B}(\mathscr{H}_{\mathscr{G}}{}^{\wedge N}, \mathscr{H}_{\mathscr{G}}{}^{\wedge M})$ invariant

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if $\mathrm{tr}\,\tau = 0$, then $\Omega, F \in \ker(T) = \mathrm{ran}(B^0)$

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$\Rightarrow \quad \{1, \det V, \overline{\det V}\} \subset \sigma(\mathscr{V}\Phi) \cap \mathbb{S}^1$

Large times asympt.

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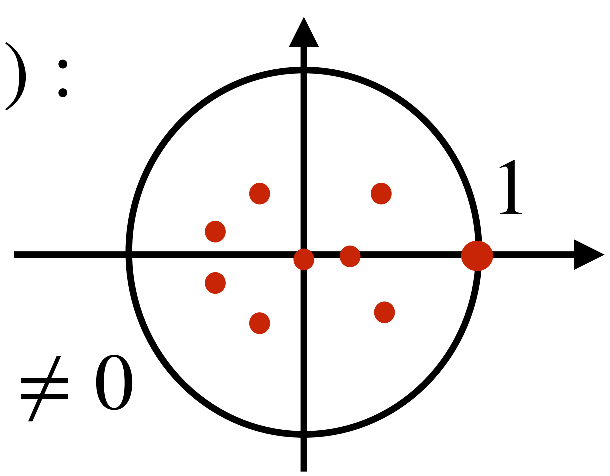
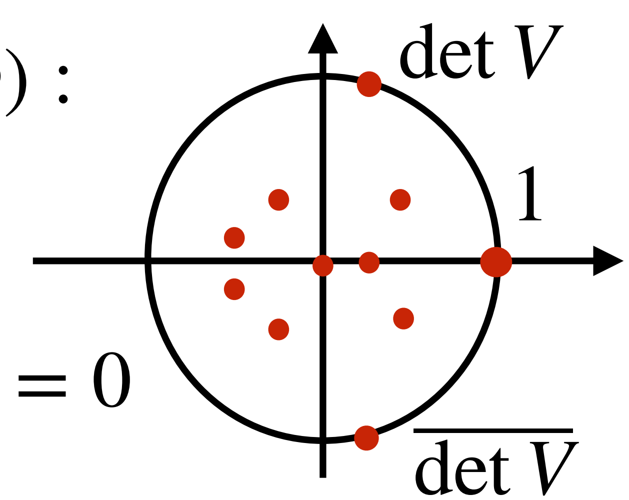


if $\operatorname{tr} \tau \neq 0$

or

$\sigma(\mathscr{V}\Phi)$:



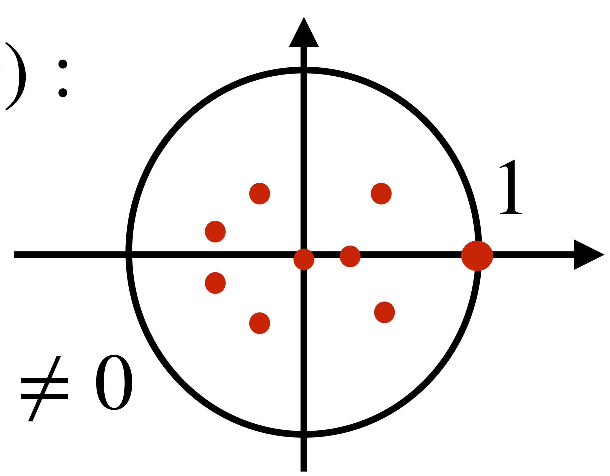
if $\operatorname{tr} \tau = 0$

Large times asympt.

Assumptions:

- $\sigma(\mathscr{V})\backslash\{1\}$ is simple, while $1 \in \sigma(\mathscr{V})$ is $\dim \mathscr{F}_-$-fold deg.
- In the eigenbasis of $\Gamma_-(V) \in \mathscr{B}(\mathscr{F}_-)$:
  - all diag. elmts of all $B^\mu,\ \mu \in \sigma(T)$, are $\neq 0$
  - each off-diag. position is $\neq 0$ in some B^μ

$\Rightarrow$ $\sigma(\mathscr{V}\Phi)$:



if $\operatorname{tr}\tau \neq 0$

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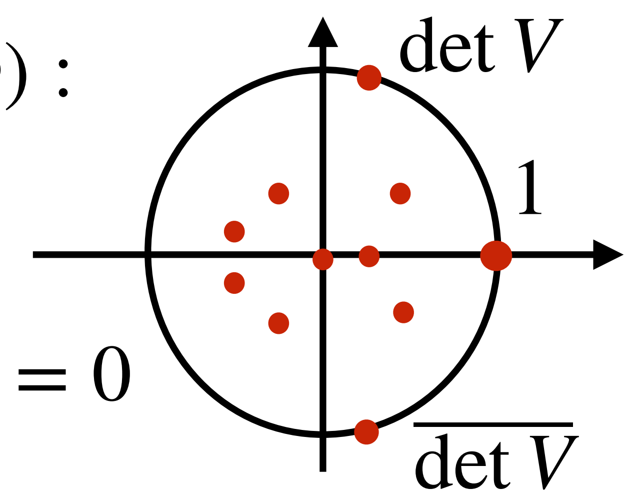

if $\operatorname{tr}\tau = 0$

$\Rightarrow$ $X \in \mathscr{P}_1 \mathscr{B}(\mathscr{F}_-)$ then $X = \sum_{0 \leq n \leq d} x_n \mathbb{1}_{\mathscr{H}_{\mathscr{G}}^{\wedge n}}$

where $\mathbb{1}_{\mathscr{H}_{\mathscr{G}}^{\wedge n}}$ proj. on $\mathscr{H}_{\mathscr{G}}^{\wedge n}$, $x_n \in \mathbb{C}$

Large times asympt.

Thm: Let $\omega_\rho(\,\cdot\,) = \mathrm{tr}\,(\rho\,\cdot)$ on $\mathscr{B}(\mathscr{F}_-)$ $(\mathrm{tr}\,\tau \neq 0)$

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Large times asympt.

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Thm: Let $\omega_\rho(\,\cdot\,) = \mathrm{tr}\,(\rho\,\cdot)$ on $\mathscr{B}(\mathscr{F}_-)$ $\quad(\mathrm{tr}\,\tau \neq 0)$

- For $\lambda > \lambda_0$ $\quad \lim\limits_{t\to\infty} \omega_\rho(\mathscr{T}_t(X)) = \omega_{\rho_\infty}(X) \qquad \forall X \in \mathscr{B}(\mathscr{F}_-)$

  where $$\rho_\infty = \bigoplus_{n=0}^{d} \binom{d}{n}^{-1} \mathrm{tr}_{\mathscr{H}^{\wedge n}}(\rho\,|_{\mathscr{H}^{\wedge n}})\, \mathbb{1}_{\mathscr{H}^{\wedge n}}$$

Large times asympt.

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- If $\rho = \rho\,|_{\mathscr{H}_{\mathscr{G}}{}^{\wedge n}}$ $\Rightarrow$ $\rho_\infty \propto \mathbb{I}_{\mathscr{H}_{\mathscr{G}}{}^{\wedge n}}$ "$\beta = 0$ Gibbs state"

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Rem: • $\mathscr{T}_t^*(\,\cdot\,) : \mathscr{D} \to \mathscr{D}$ is CPTP & unital

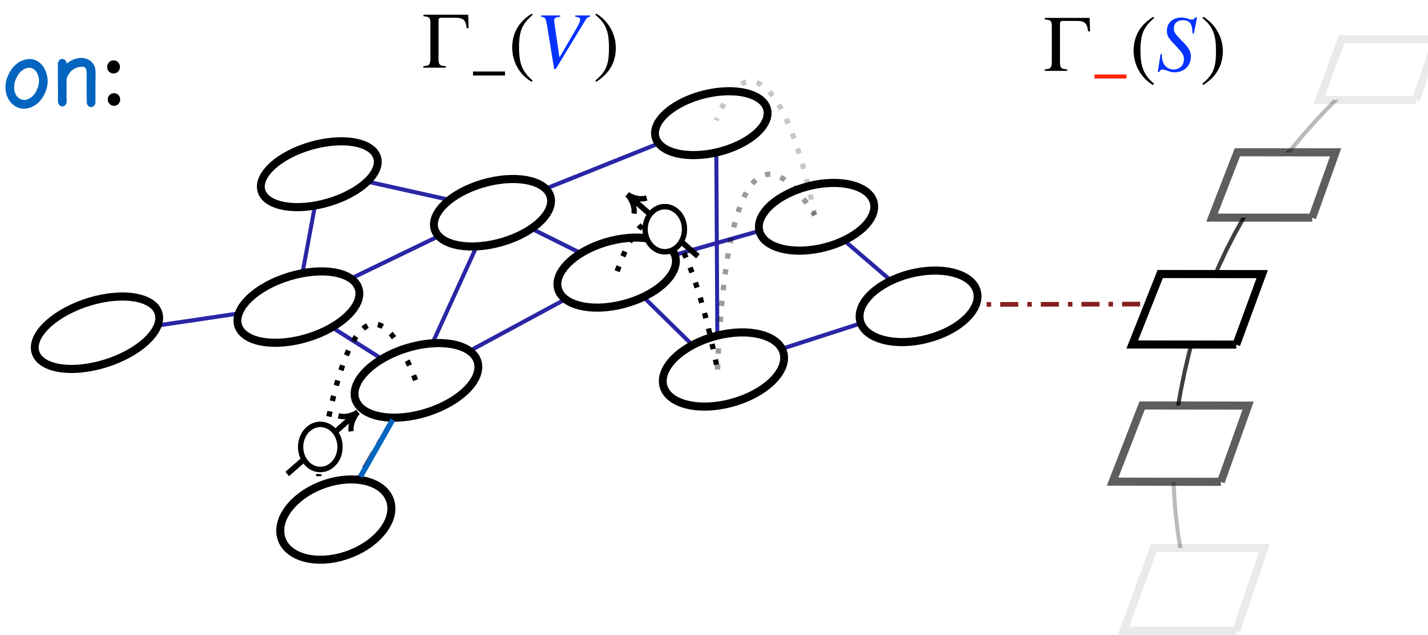
$$\Rightarrow \quad S(\rho) \leq S(\mathscr{T}_t^*(\rho)) \leq S(\rho_\infty) \leq \ln(2^d)$$

Large times asympt.

- Comparison:

fermionic

sample



Large times asympt.

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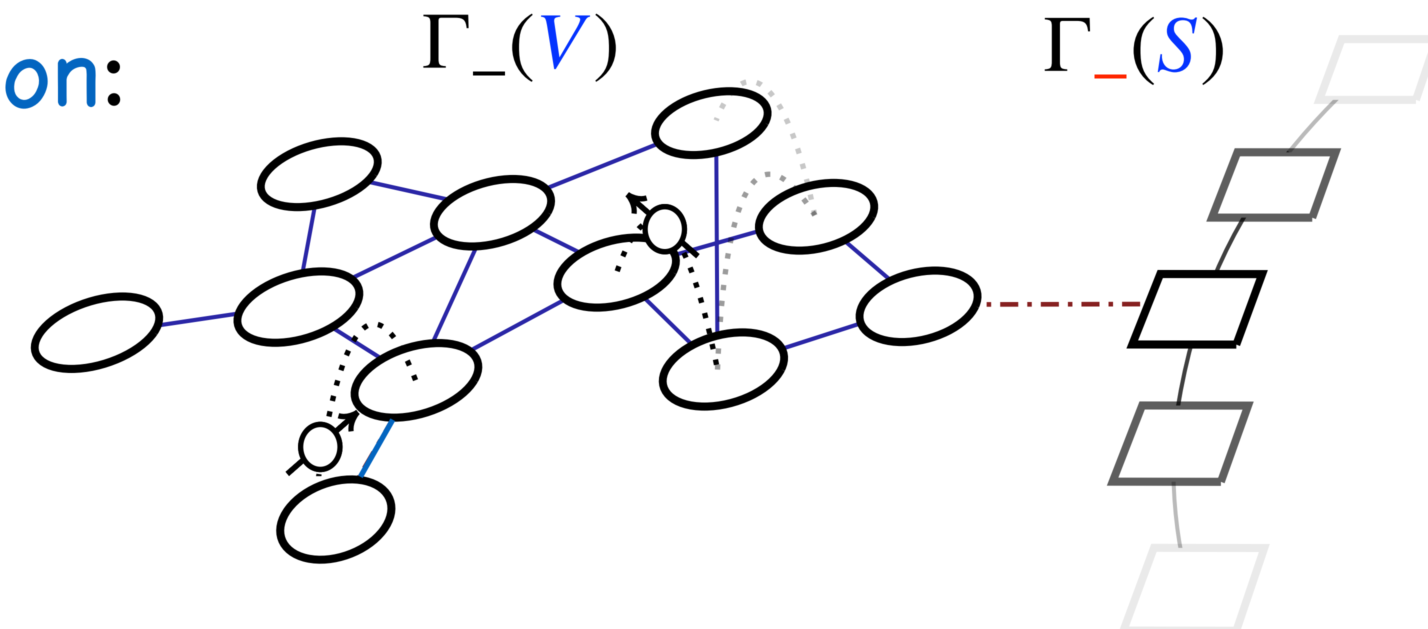
fermionic

sample



fermionic

quasifree

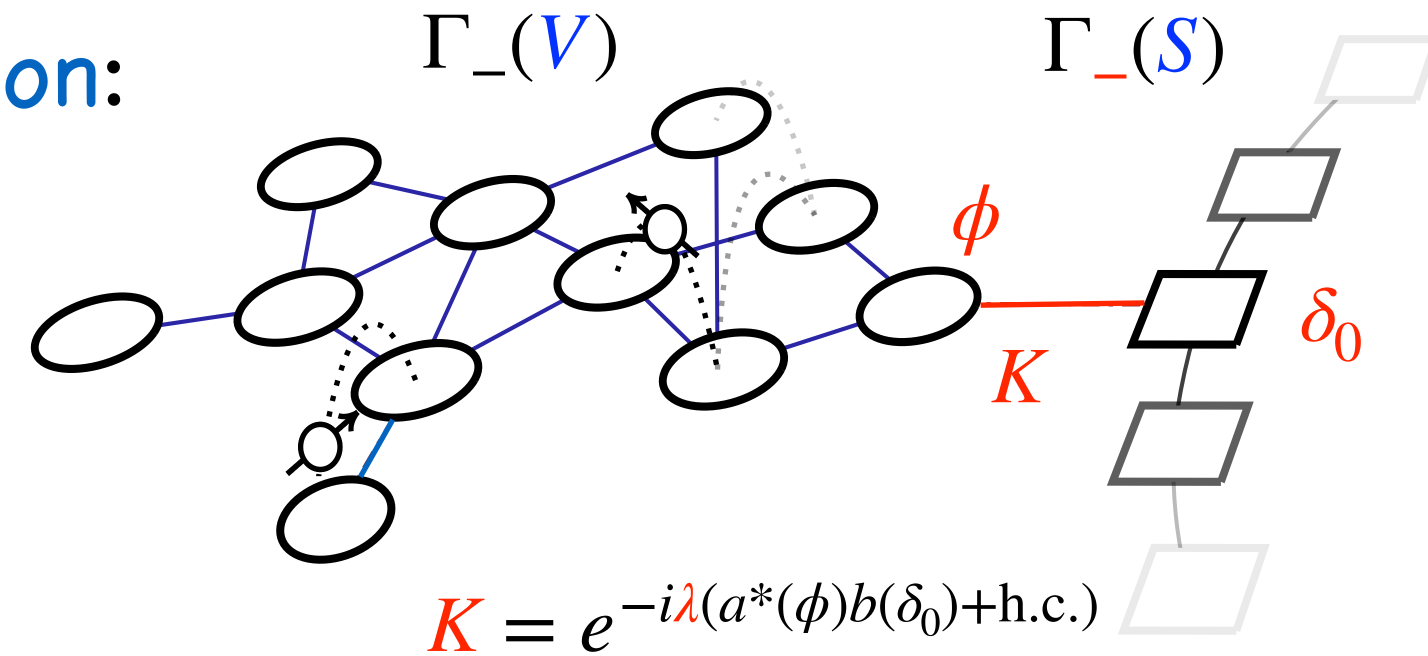
reservoir

Large times asympt.

- Comparison:

fermionic

sample



$K = e^{-i\lambda(a^*(\phi)b(\delta_0)+\text{h.c.})}$

fermionic

quasifree

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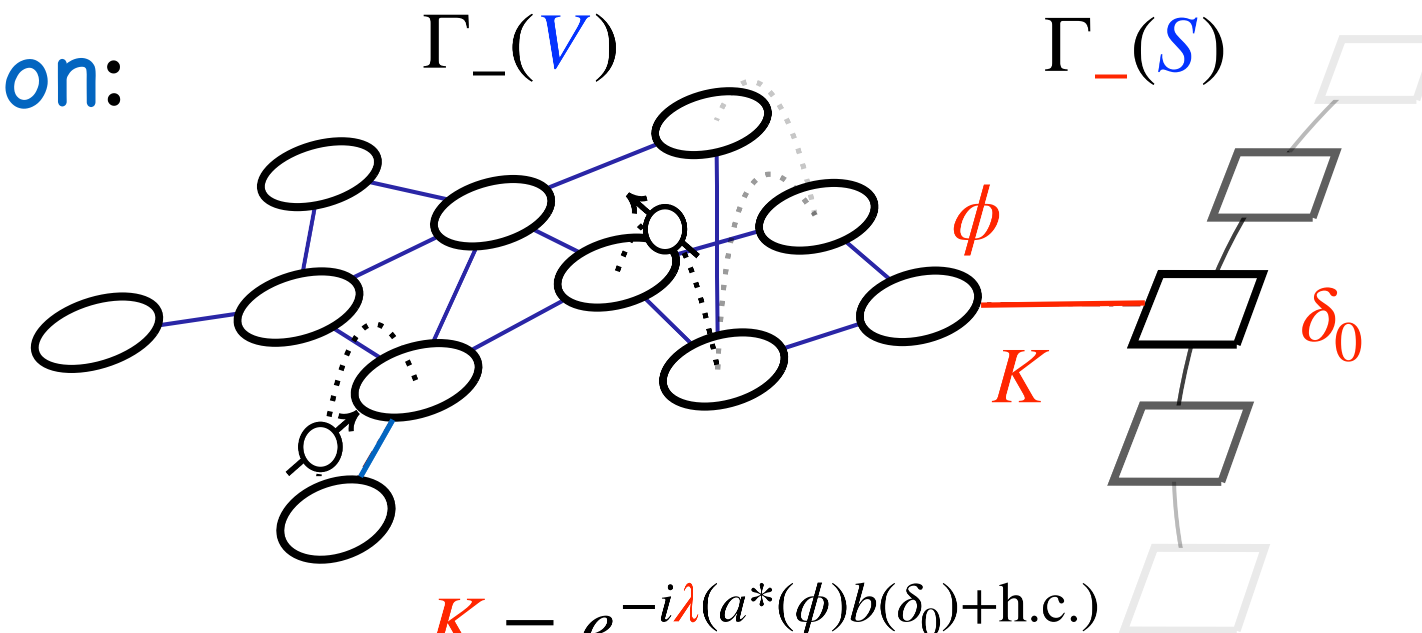
Large times asympt.

- Comparison:

fermionic sample



fermionic quasifree reservoir

$K = e^{-i\lambda(a^*(\phi)b(\delta_0)+\text{h.c.})}$

Hamza, J. '17, Raquépas '20

$\Rightarrow$ ρ_∞^Δ is quasifree, of symbol $\mathbb{0} \le \Delta \le \mathbb{1}$

Large times asympt.

- Comparison:

fermionic

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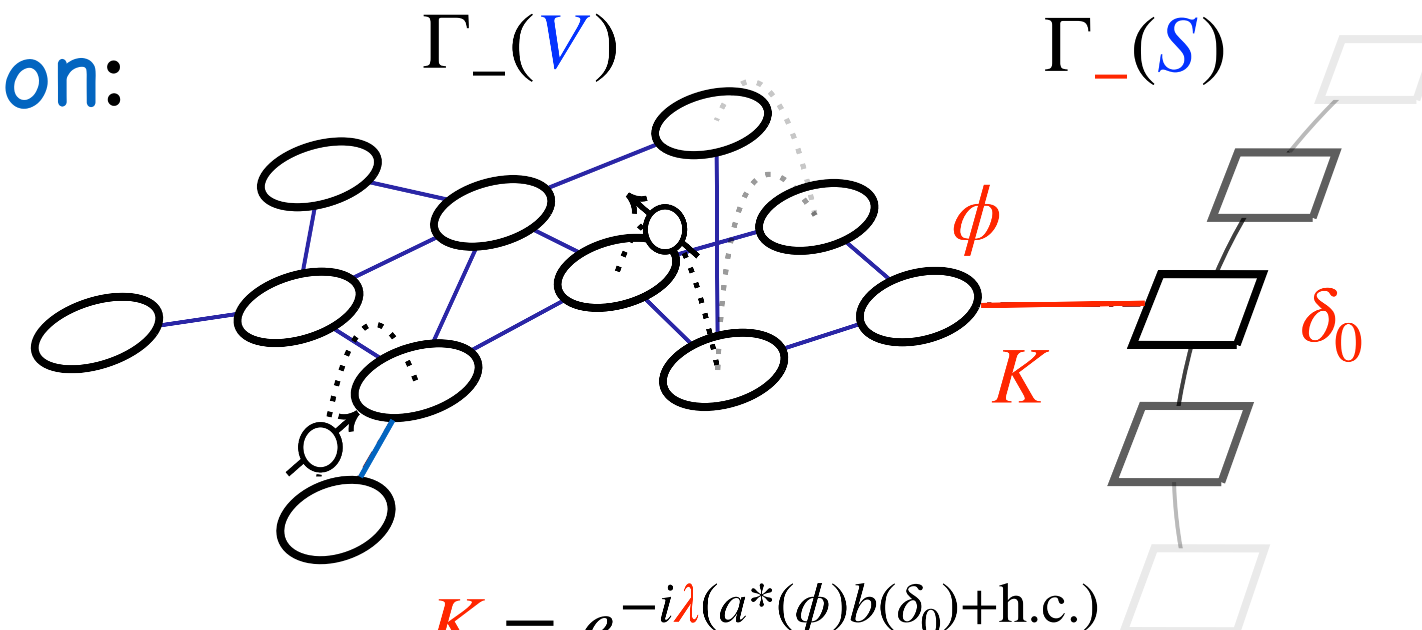

fermionic

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$K = e^{-i\lambda(a^*(\phi)b(\delta_0)+\text{h.c.})}$

Hamza, J. '17, Raquépas '20

$\Rightarrow$ ρ_∞^Δ is quasifree, of symbol $\mathbb{O} \leq \Delta \leq \mathbb{1}$ i.e.

$$\rho_\infty^\Delta = \det(\mathbb{1} - \Delta) \bigoplus_{n=0}^{d} \left(\frac{\Delta}{\mathbb{1} - \Delta} \right)^{\wedge n}$$

indep. of ρ

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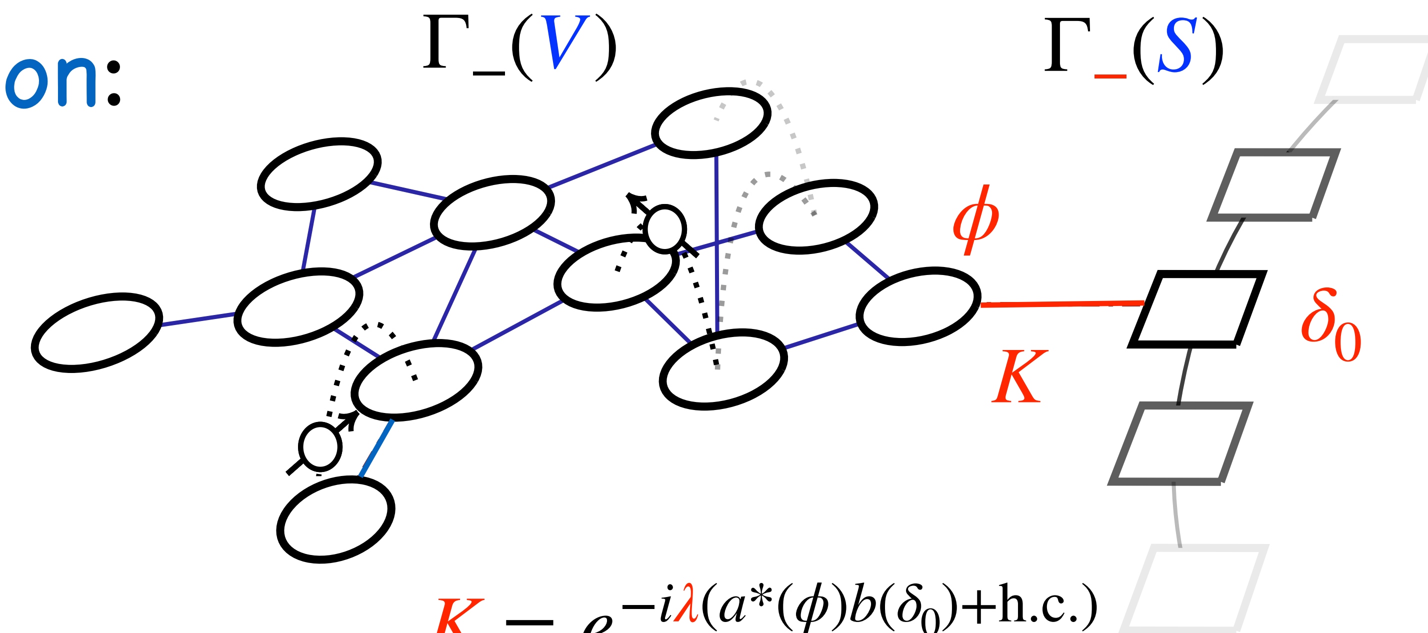
Large times asympt.

- Comparison:

fermionic

sample



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indep. of ρ

Reminder: in the bosonic case

for $\lambda > \lambda_0$

$$\rho_\infty = \bigoplus_{n=0}^{d} \binom{d}{n}^{-1} \operatorname{tr}_{\mathscr{H}^{\wedge n}}(\rho\,|_{\mathscr{H}^{\wedge n}})\, \mathbb{1}_{\mathscr{H}^{\wedge n}}$$

dep. on ρ

QW coupled to Reservoir

Thank you for your attention,

QW coupled to Reservoir

Thank you for your attention,

many thanks to John Cleese for the inspiration!