Exchangeability in Quantum Spin Systems

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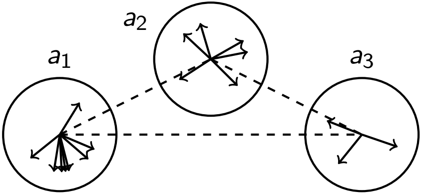
- Quantum De Finetti (applied to S_n or $S_n \times S_n$ symmetry.)

The n-partite model

- Complete n-partite graph. Relative size $a_1, a_2, \ldots, a_n$.



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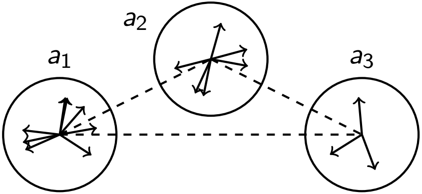

- Antiferromagnetic interaction on each edge $---$.

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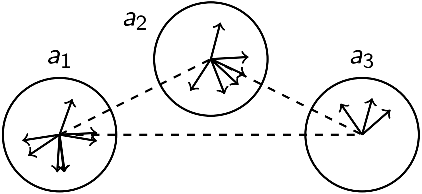

- Antiferromagnetic interaction on each edge $---$.
- Theorem: The phase transition occurs β satisfying:

$$\sum_{i=1}^{n} \frac{1}{2 - a_i \beta} = \frac{n-1}{2}$$

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Classical, binary sequence, $n = 2$. symmetric $p(x_1, x_2)$.

$$\lambda_0 = p(0,0), \quad \lambda_1 = p(0,1) + p(1,0), \quad \lambda_2 = p(1,1)$$

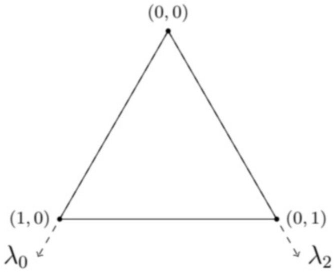
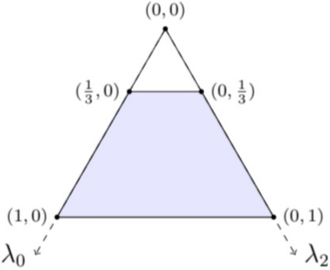
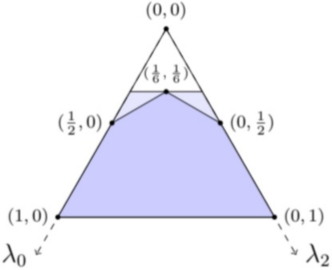


Figure 1: Symmetric distributions on $\{0,1\}^2$.

when is symmetric also exchangeable



(a) $M = 3$ shown with light blue.



(b) $M = 3, 4$ in increasing blue tone.

Figure 2: Symmetric distributions on $\{0,1\}^2$ *that can be extended to* $\{0,1\}^M$.

when is symmetric also exchangeable

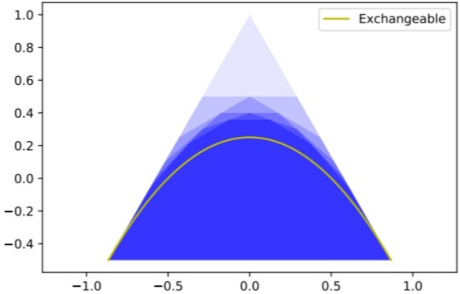


Figure 3: The marginals on $\{0,1\}^2$ *from symmetric distributions on* $\{0,1\}^M$ *for* $M = 2, 3 \ldots, 7$ *in increasing blue tone. The* $M = \infty$ *border is yellow.*

Necessary but not sufficient for general n

Conjecture: Let λ_k be probability of observing k ones in n trials. A necssary condition for exchangeablilty:

$$\lambda_0 + \lambda_n \geq \frac{1}{n}(\lambda_1 + \lambda_{n-1}) \geq \cdots \geq \frac{1}{\binom{n}{k}}(\lambda_k + \lambda_{n-k}), \quad \forall k \in [0, n/2].$$